

记住几个结论.

Page 89, 例1.  $\text{rank}(AB) \leq \min\{\text{rank}(A), \text{rank}(B)\}.$

Page 90, 例2.  $\text{rank}(A+B) \leq \text{rank}(A) + \text{rank}(B).$

Page 100, 例2. 设  $A_{m \times n} B_{n \times s} = 0$ , 则  $\text{rank}(A) + \text{rank}(B) \leq n.$

Ex.26 证明. 因为  $(A+E)(A-E) = A^2 - A + A - E = 0$ , 所以,

$$\text{rank}(A+E) + \text{rank}(A-E) \leq n.$$

另一方面,

$$\begin{aligned} \text{rank}(A+E) + \text{rank}(A-E) &= \text{rank}(E+A) + \text{rank}(E-A) \\ &\geq \text{rank}(E+A+E-A) \\ &= \text{rank}(2E) = n. \end{aligned}$$

所以,

$$\text{rank}(A+E) + \text{rank}(A-E) = n.$$